VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

CODE A

JEE Advanced Home Practice Test -3 | Paper -1 | JEE 2024

Date: 26/04/2024 **Maximum Marks: 186**

Duration : 3.0 Hours

General Instructions

1. The question paper consists of 3 Subject (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**).
   Each Part has **three** sections (Section 1,2 & Section 3).

2. **Section 1** contains **FOUR (04) Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

3. **Section 2** contains **EIGHT (08) Multiple Correct Answers Type Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** are correct.

4. **Section 3** contains **SIX (06) Numerical Value** questions. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)

5. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Name of the Candidate (In CAPITALS) : _____

Roll Number : _____

OMR Bar Code Number : _____

Candidate's Signature : _____ **Invigilator's Signature** _____

MARKING SCHEME

SECTION-1

- This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the answer. For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

SECTION-2

- This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

  Zero Marks : 0 If unanswered;

  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get –1 marks.

SECTION-3

- This section contains **SIX (06) Numerical Value** questions. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks: +3 If ONLY the correct Integer value is entered. There is **NO negative marking.**

  Zero Marks: 0 In all other cases.

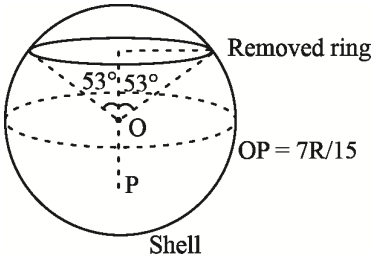
SUBJECT I : PHYSICS 62 MARKS

SECTION – 1

SINGLE CORRECT ANSWERS TYPE

This section contains 4 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or ONLY ONE Choice be Correct:

1. A thin spherical insulating shell of radius R, carries uniformly distributed charge such that the potential at its surface is V_0. A thin ring of area $\alpha 4\pi R^2 \left(\alpha << 1\right)$ is taken out from the shell without affecting rest of the shell, as shown. For the situation, which of the following options is correct?



(A) Potential at the centre of shell, after removing the ring is $V_0\alpha$

(B) Electric field at the centre of shell, after removing the ring is $\frac{V_0}{R}\left(1-\frac{3}{5}\alpha\right)$

(C) Potential at point $P\left(OP=\frac{7R}{15}\right)$ after removing the ring is $V_0\left(1-\frac{3\alpha}{4}\right)$

(D) Electric field at point P, after removing the ring is $\frac{9}{10}\left(\frac{\alpha V_0}{R}\right)$.

2. In a radioactive sample, $^{40}_{19}K$ nuclei either decay into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 9, the value of t will be: [Given : $\ln 10 = 2.3$]

(A) 9.2 **(B)** 4.6 **(C)** 1.15 **(D)** 2.3

3. A current carrying wire heats a metal rod. The wire provides a constant power $\left(P\right)$ to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature $\left(T\right)$ in the metal rod changes with time $\left(t\right)$ as $T\left(t\right)=T_0\left(1+\beta t^2\right)$

Where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of metal is:

(A) $\frac{P/2}{\sqrt{\beta T_0\left(T-T_0\right)}}$ **(B)** $\frac{P}{4\beta T_0\left(T-T_0\right)^2}$ **(C)** $\frac{P\left(T-T_0\right)^{1/2}}{\beta T_0}$ **(D)** $\frac{P\left(T-T_0\right)^{3/2}}{\beta T_0}$

4. A galaxy in the shape of a disc is rotating about its common center. The galaxy is made up of small gas molecules of mass *m*, each rotating about the common center with constant angular momentum *L*. The radius of the galaxy is *R*. Find the minimum energy required for a spaceship of mass *M* to go from radial distance $\frac{R}{2}$ to *R*.

(A) $\frac{\mu^2 L^2}{3m^2R^2}$ **(B)** $\frac{4ML^2}{5m^2R^2}$ **(C)** $\frac{3ML^2}{2m^2R^2}$ **(D)** $\frac{ML^2}{m^2R^2}$

SECTION – 2

MULTIPLE CORRECT ANSWERS TYPE

This section contains 8 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

5. Let us consider a system of units in which mass and Linear momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

(A) Dimension of acceleration is L^{-1}

(B) Dimension of volume is L^3

(C) Dimension of volumetric flow rate is L^2

(D) Dimension of velocity is *L*.

6. Two identical moving coil galvanometers have 10Ω resistance and full scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of 200 mV full scale reading and the other into an Ammeter of 10 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000\Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct?

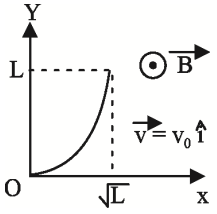
(A) Resistance of voltmeter is 100 k Ω

(B) Resistance of ammeter is approximately 2 milli-ohm.

(C) If non-ideal cell of internal resistance 10Ω is used, the measured value will decrease.

(D) Measured value of resistance is between 989 Ω to 991 Ω

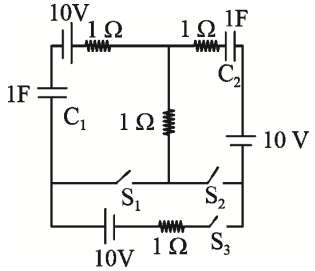
7. A conducting wire of parabolic shape, $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are :



(A) for $\beta = 1$, $\Delta\phi = \frac{B_0V_0L}{2}$ **(B)** For $\beta = 2$, $\Delta\phi = \frac{B_0V_0L}{3}$

(C) for $\beta = 0$, $\Delta\phi = B_0V_0L$ **(D)** $\Delta\phi_{\beta=\frac{1}{2}}$ is twice $\Delta\phi_{\beta=2}$.

8. For the given circuit, which of the following options are correct? (Capacitors are initially uncharged)



(A) At $t = 0,$ switch s_1 is closed, current via s_1 equals 5 A.

(B) At $t = 0$, when switch s_1 is closed, potential difference across s_2 equals 5V

(C) At steady state when all switches are closed, quantity of charge on both capacitor have different value.

(D) At steady state when all switches are closed, current via all switches equals 10 amperes.

9. A cube of side length $2a$, has volumetric charge density ρ. A Gaussian spherical surface of radius $= r,$ is taken for different situation. Which of the following option are correct?

(A) If center of cube and center of spherical surface coincide, then for $r = 1.25a,$ electric flux through spherical surface is $\frac{\rho}{\epsilon_0}\frac{13}{6}\pi a^3$

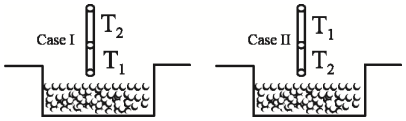
(B) If center of cube and center of spherical surface coincide, then for $r = 0.5a,$ electric flux through spherical surface is $\frac{\rho\pi a^3}{6\epsilon_0}$.

(C) If center of cube and center of spherical surface coincide, then for $r = \sqrt{2}a$, electric flux through spherical surface is $\frac{8\rho a^3}{\epsilon_0}$.

(D) If center of spherical surface is at one of the corner of the cube, then for $r = a$, electric flux through spherical surface is $\frac{\rho\pi a^3}{6\epsilon_0}$

10. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 37° and 53° respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?

[Surface tension of water $= 0.075\,N/m,$ density of water $= 1000\,kg/m^3, take\, g = 10\,m/s^2$]



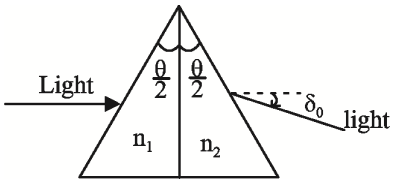
(A) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 6 cm. (Neglect the weight of the water in the meniscus)

(B) For case I, capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 6 cm. (Neglect the weight of the water in the meniscus).

(C) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be same for both cases.

(D) For case II, the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 4.5 cm. (Neglect the weight of the water in the meniscus).

11. A thin prism of angle θ, is divided into two equal parts, left side has refractive index n_1 and right side has refractive index n_2. When $n_1 = n_2 = n$, the angle of deviation equals δ_0. Refractive index can vary by $\Delta n = 10^{-3}, \Delta n << n, n = 1.5, \theta = 1°$. Given the situation, which of the following are correct?



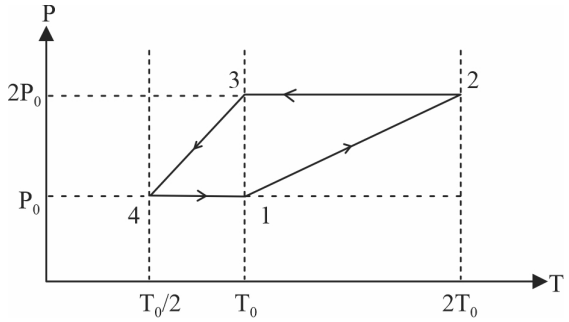
(A) If $n_1 = n - \Delta n,\ n_2 = n + \Delta n$, then $\Delta\delta$ (Change in angle of deviation) equals zero.

(B) If $n_1 = n + \Delta n,\ n_2 = n,$ then $\Delta\delta = 5 \times 10^{-4}$ degrees.

(C) If $n_1 = n, n_2 = n + \Delta n,$ then $\dfrac{\Delta\delta}{\delta_0}$ equals 1.001

(D) If $n_1 = n, n_2 = n - \Delta n,$ then $\dfrac{\Delta\delta}{\delta_0}$ equals 0.999.

12. One mole of monoatomic ideal gas goes through the given cycle (P-T) diagram. The correct statements is/are: (R is the gas constant).



(A) Work done in the whole cycle $(1 \to 2 \to 3 \to 4 \to 1)$ is $-\dfrac{RT_0}{2}$

(B) $\left|\dfrac{\Delta Q_{1-2}}{\Delta Q_{2-3}}\right| = 0.6$

(C) The whole cycle comprises of only iso-baric and adiabatic process.

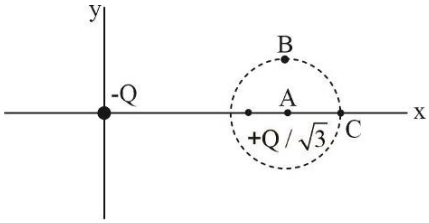
(D) $\left|\dfrac{\Delta Q_{3-4}}{\Delta Q_{4-1}}\right| = 0.6$

SECTION - 3

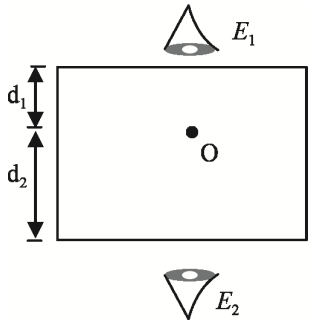
NUMERICAL VALUE TYPE

This section contains 6 Numerical Value Type Questions. For each question, enter the correct numerical value (in decimal notation, rounded-off to the second decimal place; e.g. 6.25, 7.00, –81.00, 30.27, 61.30, –7.00, –30.27, –0.33, –127.30).

13. Two-point charges $-Q$ and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0,0) and at a point (2,0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the *xy*-plane with its centre at A. All lengths are measured in meters. The point B is such that $\angle$BAC is $\frac{\pi}{2}$ and at point C circle cuts *x* axis. The *y* coordinate of B is _____*m*.



14. Refractive index of the glass slab is 1.5. There is a point object O inside the slab as shown. To eye E_1 object appears at a distance of 6 cm (from the top surface) and to eye E_2 it appears at a distance of 8 cm (from the bottom surface). Find thickness of the glass slab (in centimeters).

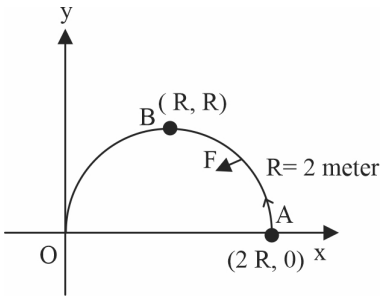


15. A small object of mass 10 kg and having some surface area is placed at the centre of a evacuated hollow spherical container. Assume that the container is maintained at 0 K. Area of container is very large as compared to the area of the body. At time $t = 0$, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2 / t_1) is _____.

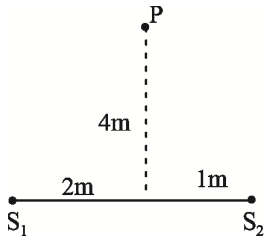
16. On *xy* plane, a force of magnitude $F = \frac{4}{r}$ Newton, directed towards origin, is given.

Where *r* is distance from origin.

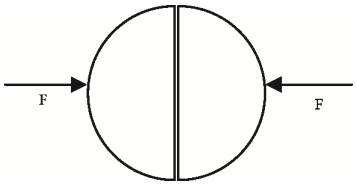
A particle moves on circular path of radius *R*, centered at $(R, 0)$. Find the work done by force while particle goes from $(2R, 0)$ to (R, R). (Given that $\ln 2 = 0.693$)



17. Two sound sources S_1 and S_2 emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s, then constructive interference occurring for various frequencies, at point *P*, given by nf_0, where $\left(n \in I^+\right)$. Find value of f_0.



18. Two metallic hemispherical shells of radius $R = 10\,cm$, contains a liquid having thermal coefficient of volumetric expansion equal to $\gamma = 10^{-6}/°C$. When liquid is shaken, the temperature of the liquid increases by $\theta = 5°C$, Bulk modulus of liquid equals $B = 5\times10^7\ N/m^2$, if the metallic shells doesn't expand, how much minimum extra force F is required to hold the liquid inside container.
(Take $\pi = 3.14$)



SPACE FOR ROUGH WORK

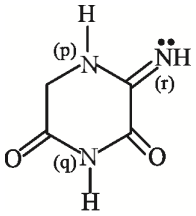
SUBJECT II : CHEMISTRY — 62 MARKS

SECTION – 1

SINGLE CORRECT ANSWERS TYPE

This section contains 4 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or ONLY ONE Choice be Correct:

19.



The correct basicity order of atoms p, q and r is:

(A) $p > q > r$ **(B)** $r > p > q$ **(C)** $r > q > p$ **(D)** $q > p > r$

20. Which of the following statements is incorrect?

(A) Cassiterite is an oxide ore of tin

(B) Tin metal is obtained by the carbon reduction of black tin (purified ore of tin).

(C) In the extraction of lead from galena, the roasting and self-reduction are carried out in the same furnace at different temperature.

(D) Reducing agent of haematite in blast-furnace is coke in upper part and CO in lower part of furnace.

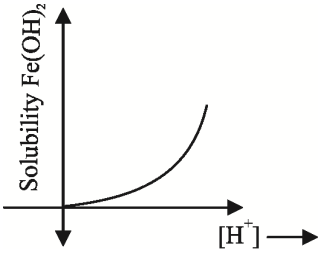
21. $K_4\left[Fe(CN)_6\right]$ can be used to detect one or more out of $Fe^{2+}, Fe^{3+}, Zn^{2+}, Cu^{2+}, Ca^{2+}$:

(A) Fe^{2+}, Fe^{3+} only **(B)** $Fe^{3+}, Zn^{2+}, Cu^{2+}$ only

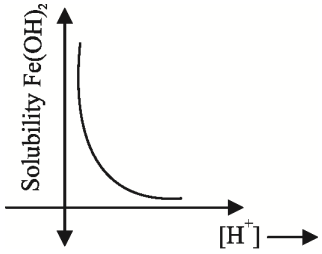
(C) All but not Ca^{2+} **(D)** All

22. Which of the following curve best represents the variation of solubility of ferrous hydroxide $Fe(OH)_2$ with the concentration of $\left[H^+\right]$ ions in the solution.

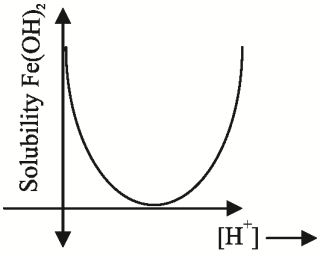
(A)



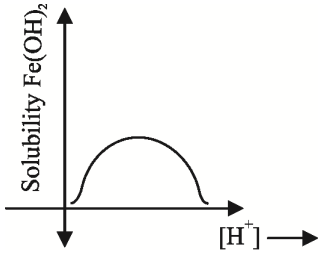
(B)



(C)



(D)



SECTION - 2

MULTIPLE CORRECT ANSWERS TYPE

This section contains 8 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

23. Which of the following statements is(are) correct?

(A) $S_2O_8^{2-}$ oxidises Ag^+ in presence of pyridine and give red colour compound.

(B) MnO_4^{2-} disproportionate to yield MnO_4^- and MnO_2 in presence of H^+ ions.

(C) In $Cr_2O_7^{2-}$ each Cr is linked to four oxygen atoms.

(D) Ti^{2+} is coloured while Ti^{4+} is colourless in aqueous solution.

24. The correct statements about anomers are

(A) Anomers have different stereochemistry at anomeric carbon atom

(B) $\alpha-$D-Glucopyranose and $\beta-D-$Glucopyranose are anomers.

(C) $\alpha-$D-Glucopyranose and $\beta-D-$Glucopyranose are diastereomers.

(D) When pure $\alpha-D-$Glucopyranose is dissolved in water its optical rotation slowly changes

25. From the following data, mark the option(s) where ΔH is correctly written for the given reaction,

Given : $H^+(aq)+OH^-(aq)\rightarrow H_2O(l);\ \Delta H=-57.3kJ$

$\Delta H_{solution}$ of $HA(g)=-70.7\,kJ/mol$

$\Delta H_{solution}$ of $BOH(g)=20\,kJ/mol$

$\Delta H_{ionization}$ of $HA=15\,kJ/mol$ and BOH is strong base.

	Reaction	$\Delta H_r\,(kJ/mol)$
(A)	$HA(aq)+BOH(aq)\rightarrow BA(aq)+H_2O$	–42.3
(B)	$HA(g)+BOH(aq)\rightarrow BA(aq)+H_2O$	–93
(C)	$HA(g)\rightarrow H^+(aq)+A^-(aq)$	–55.7
(D)	$B^+(aq)+OH^-(aq)\rightarrow BOH(aq)$	–20

26. Select correct statement for a real gas.

(A) Larger the value of $\frac{T_c}{P_c}$ of gas, larger would be the excluded volume.

(B) Critical temperature (T_c) of a gas is greater than Boyle's temperature (T_b)

(C) At critical point real gas behave as an ideal gas

(D) For a real gas $T_c=30°C$ then it is gas at 25°C and vapour at 35 °C always

27. $Ca_2B_6O_{11} + Na_2CO_3 \xrightarrow{\Delta} [X] + CaCO_3 + NaBO_2$ (unbalanced equation)

Correct choice(s) for $[X]$ is /are:

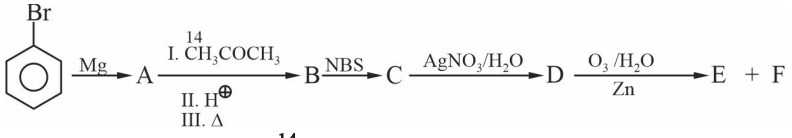
(A) Structure of anion of crystalline $[X]$ has one boron atom sp^3 hybridized and other three boron atoms sp^2 hybridized.

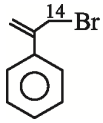
(B) X with NaOH (aq.) gives a compound which on reaction with hydrogen peroxide in alkaline medium yields a compound used as brightner in soaps.

(C) Hydrolysis of $[X]$ with HCl or H_2SO_4 yields a compound which on reaction with HF gives fluoroboric acid.

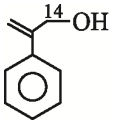
(D) $[X]$ on heating with chromium salts in oxidizing flame gives green coloured bead in cold.

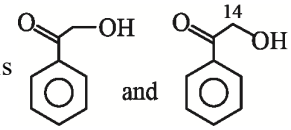
28. Consider the following sequence of reactions and identify correct option(s).



(A) Product C is 

(B) E is $CH_2{=}O$ and $^{14}CH_2{=}O$

(C) Product D is only 

(D) F is 

29. Which of the following species is/are polar in nature?

(A) XeF_4 **(B)** XeF_6 **(C)** $XeOF_4$ **(D)** BrF_5

30. Select the correct statement(s):

(A) In the reaction $^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{140}_{56}Ba + 2\,{}^{1}_{0}n + x, x \text{ is } {}^{94}_{36}Kr$

(B) In the reaction $^{23}_{11}Na + z \longrightarrow {}^{23}_{12}Mg + {}^{1}_{0}n$, the bombarding particle z is deuteron

(C) Very large amount of energy is produced during nuclear fission and nuclear fusion

(D) In a fission reaction, a loss in mass occurs releasing a vast amount of energy.

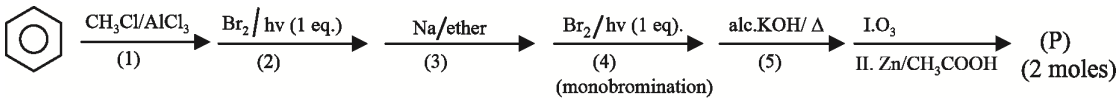
SECTION - 3

NUMERICAL VALUE TYPE

This section contains 6 Numerical Value Type Questions. For each question, enter the correct numerical value (in decimal notation, rounded-off to the second decimal place; e.g. 6.25, 7.00, –81.00, 30.27, 61.30, –7.00, –30.27, –0.33, –127.30).

31. How many of the following halides are significantly hydrolysed under the normal conditions.

$BF_3, NCl_3, NBr_3, BCl_3, PCl_3, PCl_5, CCl_4, NF_3, SiCl_4$

32. 

Find the molecular weight of 'P'.

33. An ideal solution was prepared by dissolving some amount of cane sugar (non-volatile) in 0.9 moles of water. The solution was then cooled just below its freezing temperature (271 K) where some ice get separated out. The remaining aqueous solution registered a vapour pressure of 700 torr at 373 K. Calculate the mass of ice separated out, if the molar heat of fusion of water is 6 kJ. (Answer the integer part)

34. What is the ratio of moles of $Mg(OH)_2$ and $Al(OH)_3$ present in 1 litre saturated aqueous solution of $Mg(OH)_2$ and $Al(OH)_3$ $\left(K_{sp}\text{of } Mg(OH)_2\right)=4\times10^{-12}$ and K_{sp} of $Al(OH)_3=1\times10^{-33}$. Give answer by multiplying it with 10^{-16}.

35. In how many of the following compounds of sulphur, there is $S-S$ bond (only single bond not double bond between two sulphur atoms).

$H_2S_2O_4, H_2S_2O_3, H_2S_2O_5, H_2S_2O_6, H_2S_2O_7, H_2S_2O_8, S_2Cl_2, \text{cyclic}(SO_3)_3$

36. The kinetic data for the given reaction $A(aq)+2B(aq)\xrightarrow{k}C(aq)$ is provided in the following table for three experiments at 300 K

Ex.no.	$[A/M]$	$[B/M]$	Initial rate $\left(M\,sec^{-1}\right)$
1	0.01	0.01	6.930×10^{-6}
2	0.02	0.01	1.386×10^{-5}
3	0.02	0.02	1.386×10^{-5}

In another experiment starting with initial concentration of 0.5 and 1 mole/L respectively for A and B at 300K, find the rate of reaction after 16.667 minute from start of experiment in (M/sec). Multiply answer by 10000.

SPACE FOR ROUGH WORK

SUBJECT III : MATHEMATICS 62 MARKS

SECTION – 1

SINGLE CORRECT ANSWERS TYPE

This section contains 4 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or ONLY ONE Choice be Correct:

37. For three concentric circle C_1, C_2 and C_3 with radius 1, r and 9 respectively. If from a point A on C_3 a pair of tangents to circle C_2 are drawn to touch at B and C such that BC is tangent to circle C_1, then the value of $10r$, is [where $1 < r < 9$]:

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60

38. Let $f(x)$ be a continuous function in $(0,1)$ satisfying $\int_0^1 x\sqrt{x}f(x)\left(1-\sqrt{x}f(x)\right)dx = \frac{1}{8}$. Number of solutions of the equation $f(x) = e^x$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

39. If $\frac{\cos x + \cos y + \cos z}{\cos(x+y+z)} = 2$ and $\frac{\sin x + \sin y + \sin z}{\sin(x+y+z)} = 2$, then the value of $\cos(x+y) + \cos(y+z) + \cos(z+x)$ is equal to: (where $x, y, z \in R$)

(A) 3 **(B)** 1 **(C)** 2 **(D)** -1

40. Let Z be the complex number satisfying $|z+16| = 4|z+1|$, then:

(A) $|z| = 4$ **(B)** $|z| = 5$ **(C)** $|z| = 6$ **(D)** $4 < |z| < 64$

SECTION - 2

MULTIPLE CORRECT ANSWERS TYPE

This section contains 8 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

41. If the equations $x^3 - 5x^2 + 7x - a = 0$ and $x^3 - 8x + b = 0$ have 2 common roots, then :

(A) $\log_4\left(a^3 + b^2 - 1\right)$ is equal to 2

(B) $\int_0^{\pi/2} \ln(\sin ax)\,dx = \frac{-\pi}{2}\ln 2$

(C) $\lim_{x\to 0}\left(\left[\frac{a^2 \sin x}{x}\right] + \left[\frac{b^2 \tan x}{x}\right]\right)$ is equal to 12

(D) $\tan^{-1}\left(\tan(a+b)\right)$ is equal to $6 - 2\pi$

[Note: [.] denotes the greatest integer function]

42. The ends of the major axis of ellipse are $(-2,4)$ and $(2,1)$. If the point $(1,3)$ lies on the ellipse. Then:

(A) The length of major axis is equal to 10

(B) The length of minor axis is equal to $\frac{10}{\sqrt{24}}$

(C) The length of latus rectum of ellipse is $\frac{5}{6}$

(D) Square of the distance between the foci of ellipse is $\frac{125}{6}$.

43. Let *a, b, c* denotes sides lengths of ΔABC. If *a, b, c* are the roots of $8x^3+(\lambda+2)x^2-(2k+\lambda)x-27=0$ such that $\lambda^2+2\lambda(k+1)+4k=2^3.3^5$, then which of the following is (are) correct?

(A) Circumradius of triangle ABC is $\frac{\sqrt{3}}{2}$.

(B) Distance between orthocenter and side AB is $\frac{\sqrt{3}}{4}$

(C) Distance between orthocenter and circumcentre of ΔABC is $\frac{\sqrt{3}}{4}$

(D) Distance between orthocenter and side BC is $\frac{\sqrt{3}}{2}$

44. *A* and *B* shoot independently until each shoots their target. They have probabilities $\frac{3}{5}$ and $\frac{5}{7}$ respectively of hitting the target at each shot. Then:

(A) probability that *B* required more shots than *A* is $\frac{6}{31}$

(B) probability that *B* require less shots than *A* is $\frac{10}{31}$

(C) probability that *A* and *B* require same number of shots is $\frac{15}{31}$

(D) probability that *B* require more shots than A is same as probability that *A* require more shots than *B*.

45. Let $E-ABCD$ be a pyramid on square base *ABCD* where *A* is the origin and *B* and *D* are lying on positive x-axis and y-axis respectively. If *E* is $(0,2,3)$ and $\overline{DE}.(\hat{i}+\hat{j})=0.$ then :

(A) image of the point *D* in the plane *ABE* is $\left(0,\frac{-10}{13},\frac{24}{13}\right)$

(B) image of the point *D* in the plane *ABE* is $\left(0,\frac{-6}{13},\frac{30}{13}\right)$

(C) volume of the tetrahedron *ABDE* is 2 cubic units

(D) perpendicular distance of the point *D* from the plane *ABE* is $\frac{9}{\sqrt{13}}$

46. Let $A=\begin{bmatrix} a & b & 1 \\ 2 & 1 & 3 \\ 1 & c & 2 \end{bmatrix}$ and $A^{-1}=\frac{1}{3}\left(5A-A^2\right)$, then:

(A) $|A|=3$ **(B)** $|A|=-3$ **(C)** $Tr(A)=5$ **(D)** $Tr(A)=a+b+c$

47. If $f(x)=x^2+xg'(1)+g''(2)$ and $g(x)=f(1)x^2+xf'(x)+f''(x)$. Then:

(A) minimum value of $f(x)$ is equal to -2.25

(B) the value of $\int_2^3 \frac{dx}{f(x)-x+5}$ is equal to $\frac{\pi}{4}$

(C) number of positive integral values in the domain of $\sqrt{\frac{f(x)}{g(x)}}$ is 4

(D) number of points where $g(|x|)$ is non derivable is 1.

48. Let $f(x)$ be a continuous function defined for every real $x \in R$. For any real numbers 'a' and 'b' that satisfy $a<b$, $f(x)$ always satisfies $f(a)>f(b)$. Then which of the following is/are correct ?

(A) $\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}$ exists and negative

(B) There is always only one real root of $f(x)=0$

(C) There is always only one real root of $f(x)=f(-x+1)$

(D) There is no real root of $f(x)=f(x+1)$

SECTION - 3
NUMERICAL VALUE TYPE

This section contains 6 Numerical Value Type Questions. For each question, enter the correct numerical value (in decimal notation, rounded-off to the second decimal place; e.g. 6.25, 7.00, –81.00, 30.27, 61.30, –7.00, –30.27, –0.33, –127.30).

49. Let E and M be 3×3 matrices satisfying the system of equations

$$EM^T=(EM)^T=20I$$

And $(E+M)^T=17(E-M)^T$

Where I denotes identify matrix of order 3.

If $\left(E^2+M^2\right)=\frac{a}{b}I$ (where a and b are co-prime), then find the value of $(a+b)$.

50. $\vec{a},\vec{b},\vec{c}$ are 3 non-coplanar unit vectors inclined at an angle $\alpha\left(\le\frac{\pi}{2}\right)$ to each other. If the volume of tetrahedron formed by these vectors is $\frac{1}{\sqrt{360}}$, then find the value of $10\left(3\cos^2\alpha-2\cos^3\alpha\right)$.

51. Find the value of $S=\frac{2+6}{4^{100}}+\frac{2+2(6)}{4^{99}}+\frac{2+3(6)}{4^{98}}+.....+\frac{2+99(6)}{4^2}+\frac{2+100(6)}{4}$

52. Let f be a continuous and even function such that $\int_0^a f(x)dx=10$. If $g(x)$ is a continuous positive function such that $g(x)g(-x)=1$ and $\int_0^a g(x)dx=5,$ then find the value of $\int_{-a}^{a}\frac{f(x)}{1+g(x)}dx$.

53. Let A and B be two sets of complex numbers such that $A=\left\{z:\left(|z|^3-2|z|^2+3i|z|-6i=0\right)\right\}$ and $B=\left\{z:\left(\left|z^2-4\right|+|z^2+4|\le 4|z|\right)\right\}$. Find the area of the figure enclosed by joining the points lying in $A\cap B$.

54. Let line L_1 be the reflection of a tangent to the parabola $(y-2)^2=4(x-1)$ drawn at P in the line $x=1$. If area of the triangle formed by the line L_1, the tangent and straight line $y=2$ is 64 sq. units, then find the abscissa of point P.

SPACE FOR ROUGH WORK